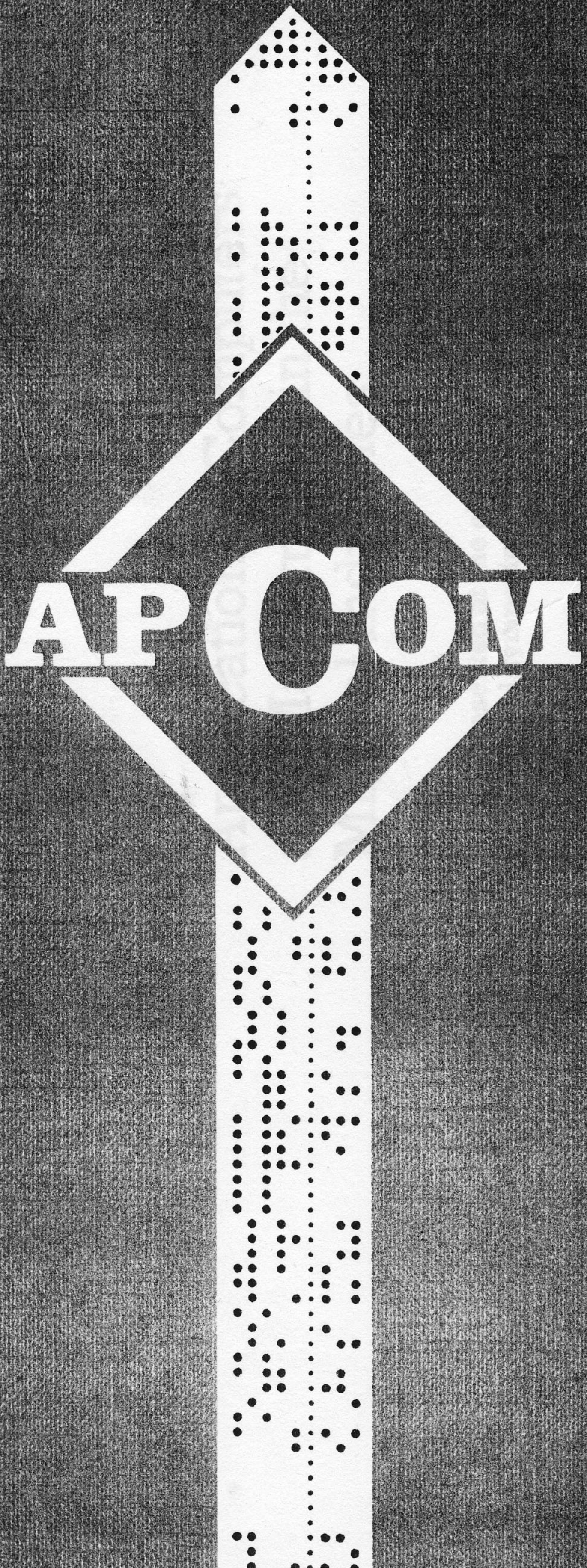

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APPLICATION OF COMPUTERS AND MATHEMATICS IN THE MINERAL INDUSTRIES

Institution of Mining
and Metallurgy

Improvement of interpretation of uranium leaching tests by inclusion of geological information

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SYNOPSIS

Correspondence Analysis[1] has been applied in the last decade to geological problems[2-3-4] and to mine planning[5]. An extension of this method, denoted *qualitative regression*[6], is used in this paper for the purpose of incorporating qualitative geological information into a black-box model of a leaching process of Uranium ores. This model relates the recovery of the process to a set of relevant qualitative and quantitative "explicative" geological variables, the contribution of which to recovery being also assessed through Correspondence Analysis. The model can be used to identify *targets* for geological exploration (evaluating their favorability in terms of expected recovery); moreover, it allows the planning of further processing tests, minimizing their number and making them consistent with geological environment of samples delivered to the metallurgist.

AVAILABLE DATA

Fifty DDH samples of Uranium ore were collected in three metasediment Iberian type deposits located in an area of about 3 Km^2, with the same broad regional geology. As expected, local particularities are found in detailed small scale investigation. The following geological features appear as significant: the mineralization occurs sometimes as a fine dissemination or as macroscopic minerals; the orebodies are located inside or outside a metamorphic aureole and the mineralization may be associated to brecciated zones. This paper is concerned to these features, considered in their regional sense.

For each sample, the following in situ attributes were recorded:

- . Presence of quartz
- . Presence of a fault
- . Presence of expressed mineralization
- . Presence of contact metamorphism
- . Depth from topographic surface
- . Lithology
- . Redox Condition

Except for the attribute Depth, all the others are discrete two or three state variables (*categorial data*), which may control, to some (unknown) extent, the leaching process recovery.

In the same operational conditions, each sample was submitted to a laboratory leaching test, in order to assess the influence on recovery of different in situ attributes.

The leaching tests were performed in the following operational conditions:

- . Amount of acid (H_2SO_4) - 40 Kg/t
- . % of solids in the pulp - 60 %
- . Amount of oxidant (MnO_2) - 3 Kg/t
- . Residence time - 12 h
- . Ambient temperature

For each test, the U_3O_8 feed grade was measured, as well as the grade in the residue. The recovery is simply defined as the complement to 1 of the ratio: residue grade/feed grade (expressed as %).

CODIFYING THE VARIABLES

In order to ensure homogeneity for the variable set, histograms for continuous variables (depth, feed grade and recovery) were calculated, classes of similar frequence were obtained, and boolean codes were assigned to those classes.

After this procedure (performed according to the rules of *complete disjunctive code*[1]) each test is characterized by a set of 9 variables, each one of them taking the value one for the state where there is occurrence, and value zero otherwise.

In Table 1 are listed all possible states taken by the set of variables.

Table 1 Codes for the states taken by variables

VARIABLE	POSSIBLE STATES		
Presence of Quartz	YES $(Qz)_1$	NO $(Qz)_0$	
Presence of Fault	YES $(FAULT)_1$	NO $(FAULT)_0$	
Presence of Expressed Mineralization	YES $(MINERALIZATION)_1$	NO $(MINERALIZATION)_0$	
Presence of contact metamorphism	YES $(METAMORPHISM)_1$	NO $(METAMORPHISM)_0$	
Depth D (m)	$D<14.15$ D_1	$14.15 \leq D \leq 36.70$ D_2	$D>36.70$ D_3
Lithology	SHALE	TRANSITION	SANDSTONE
Redox Condition	REDUCED	TRANSITION	OXIDIZED
Feed Grade F ($\%U_3O_8$)	$F<.09$ F_1	$.09 \leq F \leq .20$ F_2	$F>.20$ F_3
Recovery R (%)	$R<63.89$ R_1	$63.89 \leq R \leq 85.00$ R_2	$R>85.00$ R_3

SELECTION OF THE RELEVANT VARIABLES

In order to choose those variables that provide an important contribution to the recovery, and disregard redundant ones, a Correspondence Analysis was carried out, taking as input a square multidimensional contingency table (Burt Matrix, as denoted by Benzécri[1]), each cell of which representing joint occurrences of all possible combinations of states taken by the set of variables given in Table 1.

As a result of this Correspondence Analysis, it is shown in Fig. 1 the projection of all states on the two principal axes.

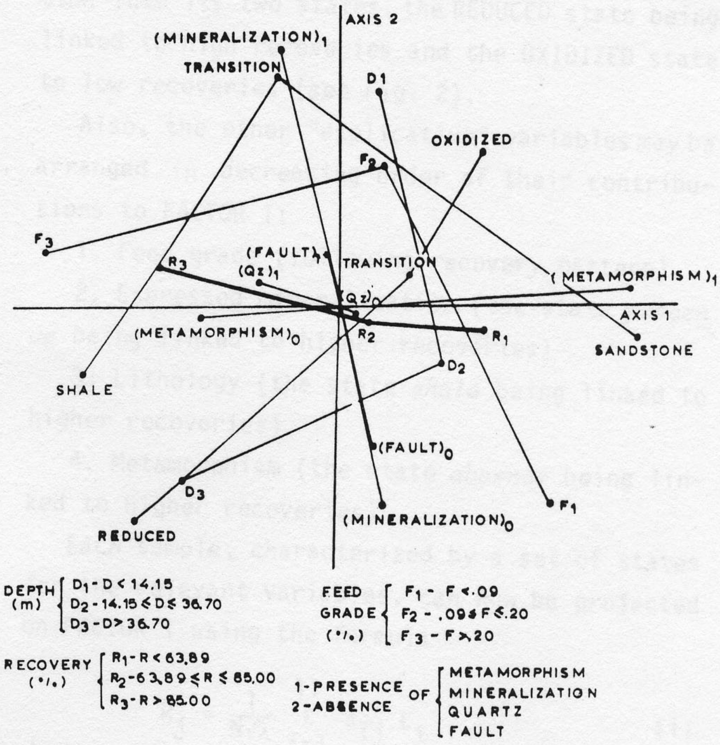

Fig. 1 - Projection of states on the principal axes obtained from Correspondence Analysis (Burt Matrix)

The interpretation of Fig. 1 leads to the following conclusions:

. The presence or absence of *Quartz* has a neglegible influence on the variable recovery (the projections of $(Qz)_0$ and $(Qz)_1$ are plotted near the origin).

. The variable *Depth* exhibits a variation pattern similar to that of the *Redox Condition* (compare line $D_1 \rightarrow D_2 \rightarrow D_3$ to OXIDIZED → TRANSITION→ → REDUCED).

. The variable *Presence of Fault* follows the *Expressed Mineralization* pattern (lines $(FAULT)_1 \rightarrow$ $\rightarrow (FAULT)_0$ and $(MINERALIZATION)_1 \rightarrow (MINERALIZATION)_0$).

Once these conclusions checked against geological evidence, the decision was taken to reject variables *Presence of Quartz, Depth* and *Presence of Fault* and proceed further analysis with the remaining variables.

QUALITATIVE REGRESSION

In order to visualize how recovery is linked to the states for the "explicative" variables, a new set of five contingency tables was constructed. Each contingency table presents absolute frequencies put in the cross-tabulation form of *'states of recovery by states of each one of the relevant variables'*

For the sake of simplicity (and taking into account their poor geological meaning), the few occurrences of transition states for variables *Lithology* and *Redox Condition* (9 and 6) where randomly distributed to one of the other states of the same variable.

So, the set of contingency tables has now 3 rows × 11 columns.

Using this set of contingency tables as an input, the Correspondence Analysis program was run, leading to the projection of the states for relevant variables on two factors, which account for the totality of the inertia of the cloud (as the input matrix has 3 rows, two factors explain 100% of the inertia of the cloud).

In Fig. 2 are plotted the projection of the states for relevant variables on the two factors arising from Correspondence Analysis.

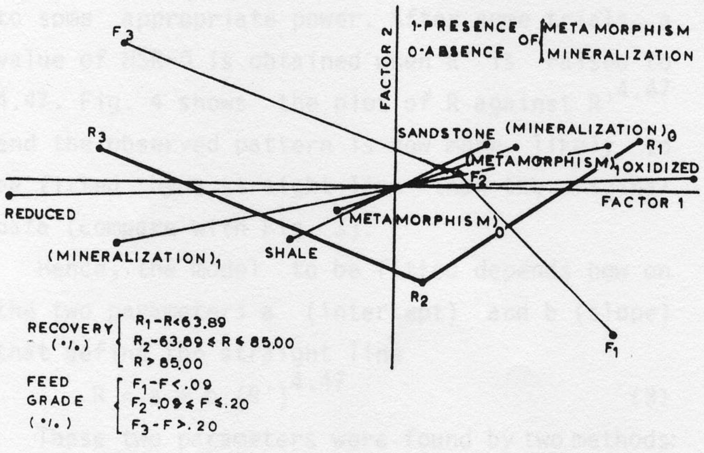

Fig. 2 - Projection of states for relevant variables on the two FACTORS arising from Correspondence Analysis (3 × 11 set of contingency tables)

The variation pattern of the states for recovery ($R_1 \rightarrow R_2 \rightarrow R_3$) shows that this variable may be almost fully explained by FACTOR 1 (which accounts for 92.45 % of the inertia of the cloud). Therefore it was decided to take FACTOR 1 as a "synthesis" of explicative variables and look for its interpretation in terms of these variables.

In a first approach, FACTOR 1 may be interpreted as the axis that splits variable *Redox Condition* into its two states, the REDUCED state being linked to high recoveries and the OXIDIZED state to low recoveries (see Fig. 2).

Also, the other "explicative" variables may be arranged in decreasing order of their contributions to FACTOR 1:

1. Feed grade (following recovery pattern)
2. Expressed mineralization (the state *presence* being linked to higher recoveries)
3. Lithology (the state *shale* being linked to higher recoveries)
4. Metamorphism (the state *absence* being linked to higher recoveries)

Each sample, characterized by a set of states for the relevant variables, can now be projected on FACTOR 1 using the formula

$$R'_j = \frac{1}{N\sqrt{\lambda}} \sum_{i=1}^{11} \delta_{ij} L_i \qquad (1)$$

where

$i = 1,11$ is the state for variables

$j = 1,50$ is the sample ordinal

R'_j is the projection of sample j on FACTOR 1

N is the number of variables (5)

λ is the eigenvalue for FACTOR 1

δ_{ij} is a boolean code that takes the value 1 if the state i occurrs for sample j, and 0 otherwise

L_i is the loading of state i on FACTOR 1

As the projections R' depend only on the states for relevant "explicative" variables, it is now possible to attempt the search for a black-box model relating the recovery to those states.

Omitting subscript j, the model is

$$R = \psi(R')$$

$$R = \psi\left(\frac{1}{N\sqrt{\lambda}} \sum_{i=1}^{11} \delta_i F_i\right) \qquad (2)$$

where

R is the recovery

ψ is some relationship (to be found) linking R to R'

MODEL FITTING

The simple plot of R against R' suggests a certain relationship between these two variables (Fig. 3). Nevertheless, the shape of that relationship does not appear to be linear. Indeed, if the median of each third of data is found (points M_1

M_2 and M_3 in Fig. 3), the slope of the straigh line passing through M_1 and M_2 is very different from the corresponding slope for points M_2 and M_3.

The Half Slope Ratio (ratio between slopes of lines $\overline{M_2M_3}$ and $\overline{M_1M_2}$) measures how straight the relationship between R and R' is[8]. In the case of Fig. 3, the Half Slope Ratio (HSR) is 2.60. Such a value for the HSR criterion indicates that re-expression of data is required, if a linear fit is to be met.

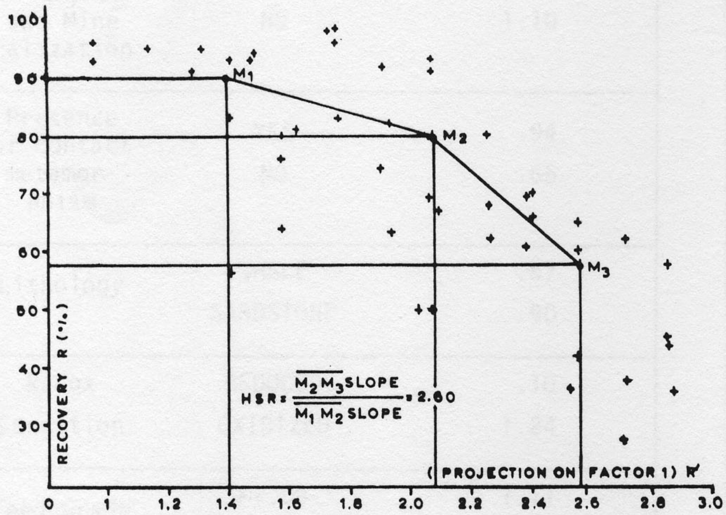

Fig. 3 - Plot of recovery against the projection of samples on FACTOR 1

The re-expression assumed to reshape the plot into a straight line is to raise each data point to some appropriate power. After some trials, a value of HSR=0 is obtained when R' is raised to 4.47. Fig. 4 shows the plot of R against $R'^{4.47}$ and the observed pattern is now more likely to be fitted by a straight line than the original data (compare with Fig. 3).

Hence, the model to be fitted depends now on the two parameters a (intersept) and b (slope) that define the straight line

$$R = a + b\,(R')^{4.47} \qquad (3)$$

These two parameters were found by two methods: least squares regression and resistant line fit[8]. Results of the two methods are given in Table 2.

Table 2 Results of linear fit

	RESISTANT LINE	LEAST SQUARES
Intersept (a)	91.67	87.95
Slope (b)	-.49	-.44
Correlation Coeff.	-.76	-.76
Residuals' slope	.00	-.04
Residuals'variance	159.04	156.67

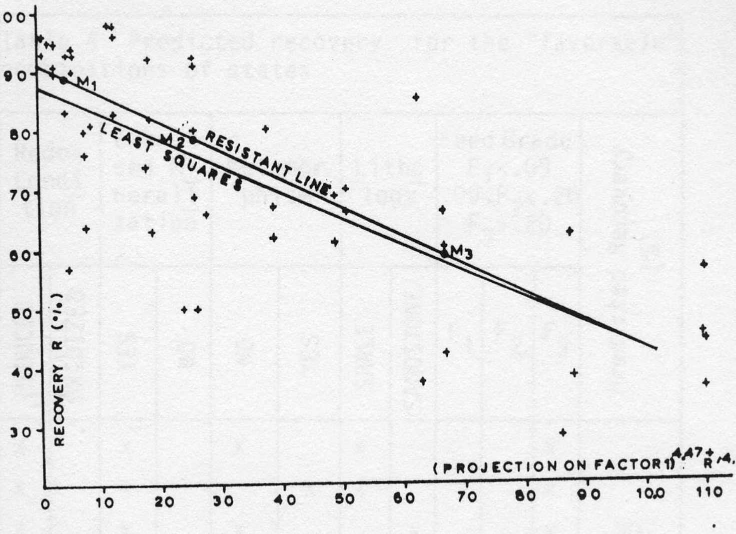

Fig. 4 - Model fitting to the plot of recovery against re-expressed projection of samples on FACTOR 1

As there is no significant improvement on the variance explained by the model when least squares is used instead of resistant line, it seems advisable to use the later fit, which is less sensitive to outliers and ensures uncorrelated residuals. Furthermore, no reliable confidence intervals may be calculated from least squares regression, as Gaussian conditions are far to be met (apart from other considerations discussed in the sequel).

Noting that the number of three states was fixed a priori for the variable recovery, it is necessary, at this state, to evaluate how the model is sensitive to changes in limits of the classes defining the states for recovery. Using two and four states and repeating the whole analysis, the conclusion was reached that the correlation coefficient decreases in 6.20 % and 2.56 %, respectively.

Also, a double regression using projections on factors 1 and 2 confirmed that factor 2 does not contribute (in centesimal precision) to explain the recovery variability.

Rearranging equation (2) and introducing parameters of equation (3), the final model is written as

$$R = 91.67 - .03\,\Big(\sum_{i=1}^{11} \delta_i\, L_i\Big)^{4.47} \qquad (4)$$

Values of loadings L_i are given in Table 3 for each state of the relevant variables.

Table 3 Loadings for states where variables occur

VARIABLE	STATE	LOADING L_i
Presence of Expressed Mineralization	YES	.28
	NO	1.10
Presence of contact metamorphism	YES	.94
	NO	.65
Lithology	SHALE	.57
	SANDSTONE	.90
Redox Condition	REDUCED	.10
	OXIDIZED	1.24
Feed Grade F (% U_3O_8)	$F<.09$	1.11
	$.09 \leq F \leq .20$	.85
	$F>.20$	.19

Using L_i values from Table 4, it is straigtforward to predict the expected recovery (before any leaching test is performed) for a new sample (characterized by a set of eleven boolean codes that take five values 1 where there is occurrence of a certain particular state for the five relevant variables, and 0 otherwise).

For the set of the most "favorable" states for relevant variables (presence of expressed mineralization, absence of contact metamorphism, shale, reduced, highest feed grade), a maximum of 91.27 % is reached for the predicted recovery, applying model (4) to the appropriate loadings given in Table 3.

Conversely, for the less "favorable" states (those presenting higher loadings) a minimum of 40.27 % is obtained from (4).

For other values of the predicted recovery, the states for relevant variables are balanced through each particular set of loadings, as summarized in Table 4 (for expected recoveries greater than 85 %).

Table 4 Predicted recovery for the "favorable" combinations of states

Redox Condition		Expressed Mineralization		Metamorphism		Lithology		Feed Grade $F_1<.09$ $.09 \leq F_2 \leq .20$ $F_3>.20$			Predicted Recovery (%)
REDUCED	OXIDIZED	YES	NO	NO	YES	SHALE	SANDSTONE	F_1	F_2	F_3	
X		X		X		X				X	90
X		X			X	X				X	
X		X		X			X			X	
X		X			X		X			X	
X		X		X		X			X		
X			X	X		X				X	89/90
X		X		X		X		X			
X		X			X	X			X		88/89
X		X		X			X		X		
X			X		X	X				X	
	X	X		X		X				X	
X			X	X			X			X	87/88
X		X		X		X		X			
X		X		X			X	X			
X		X			X		X		X		
	X	X			X	X				X	86/87
X			X		X		X			X	
	X	X		X			X			X	85/86
X			X	X		X			X		
X		X			X		X	X			

A practical direct application of Table 4 is to indicate expected profitable targets for further exploration in the area where the orebodies are located. It is worth to emphasize that these targets have a stronger economic meaning than the usual "anomalies". Indeed, the "favorable" combinations of geological states can be assessed quantitatively and expressed in terms of expected recovery. An obvious analysis of Table 4 contributes to the delineation of an "ideal" exploration target (defined here as to yield predicted recoveries greater than 85 %). Indeed, the frequency of favorable geological states *Reduced, Ex-*

pressed mineralization, *absence of metamorphism* and *shale* are respectively, 85 %, 75 %, 60 % and 55 %.

DISCUSSION OF THE METHOD

The principal merit of the method applied in this case study is that a distribution-free model with geological meaning was found to relate quantitative variables to categorial data. Other methods dealing with categorial data (as some kind of analysis of variance or log-linear models) present two drawbacks: they are cumbersome to apply to a 9 order multidimensional table (which is a moderately high order) and are based on strong assumptions about data probability density functions. These assumptions are seldom met in Earth Sciences data and hard to check.

The distribution-free characteristic of the method applied in this case study (which is based mainly on *geometric* considerations) has a price to be payed - the difficulty to find reliable confindence intervals for the model (that explains only 58 % of the variability for recovery). As a matter fact, the approximations and shortcuts made for qualitative regression through Correspondence Analysis are supported both by geological evidence and geometric considerations; on the other hand, no goodness of fit test is provided for the resistant line. Even jacknifing the samples, it is obtained an underestimated variance for the "response" variable, because it is unrealistic to assume independence for sucessive realizations of variable R' (apart from spatial dependence, it must be noted that R' is obtained through equation (1)).

Therefore the advantages of the method rely mainly on the Correspondence Analysis straighforward ability to depict graphically the variables and to suggest a relationship between recovery and certain geological attributes. That relationship makes sense (from a geological and ore processing stand-point) but the reliability of its estimation can not be established (and so, no probabilistic bounderies can be assigned to the unexplained 42 % of the "response" variability). This fact is also due to the small number of samples available.

CONCLUSIONS

Once set up the limitations of the method - and apart from representativity problems regarding spatial location of samples in the orebodies -, it can be stated that the case study described in this paper illustrates an effective device to model the interactions between geological attributes and ore processing economic variables.

This link between geology and ore processing (expressed, in quantitative terms, for each sample delivered to the metallogist) is an important feature to improve the global productivity of a mine venture.

Regarding the actual and potencial applications of the model, it is worth noting that it can be used in the main stages of developement of a new mine:

. *In Exploration*, the model allows the characterization of geological targets with greater economic meaning than the usual "anomalies".

. *In Mine Planning*, the model (when adjusted to local particularities of each orebody and scaled up to industrial recoveries) may be introduced into the production scheduling scheme, improving the usual mining selection (based on feed grades) with information about expected recoveries.

. *In Ore Processing*, the planning of further tests can be improved, focusing the attention of the metallurgist to more "difficult" ore (the importance of which in the orebody can be evaluated, if sampling is assumed as "representative").

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